Ravne rotacione krive spiralnih galaksija objasnjene konstantnom brzinom nelinearnih talasa gustine ili Da li nam treba tamna materija na malim skalama?

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Astronomska opservatorija Beograd

1. Nelinearni talasi gustine kao resenje Nelinearne Sredingerove jednacine

2. Izraz za rotacionu brzinu zvezdane komponente spiralnih galaksija

3. Primena relacije rotacione brzine na Mlecni put i nekoliko tipicnih i netipicnih spiralnih galaksija

4. Zakljucak i buduca istrazivanja

MF BU, Beograd, Oktobar 2021

Nelinearni talasi gustine kao resenje Nelinearne Sredingerove jednacine

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \varphi}(\rho v_\varphi) = 0$$

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_r}{r}\frac{\partial v_\varphi}{\partial \varphi} - \frac{v_\varphi^2}{r} = -\frac{\partial \phi}{\partial r}$$

$$\frac{\partial v_\varphi}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_r v_\varphi}{r} = -\frac{1}{r}\frac{\partial \phi}{\partial \varphi}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \varphi^2} + \frac{\partial^2 \phi}{\partial z^2} = 4\pi G\rho,$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = -4\pi G\sigma(r,\theta)\,\delta(z).$$

aproksimacija beskonacno tankog diska:

model koji su predlozili Lin i Shu; gustina u z-pravcu zamenjena delta funkcijom; Poasonova jednacina

(Lin and Shu, 1964)

$$\frac{\partial \phi(r, z=0)}{\partial r} = \pm 2\pi \mathrm{i} G\sigma$$

$$\sigma = -\frac{k}{2\pi G}\phi(z=0),$$

$$k = -\frac{\mathrm{i}}{\phi}\frac{\partial \phi}{\partial r}$$

Reduktivna perturbaciona metoda (Jaffrey A. i Kawahara T. 1982; Jaffrey A. i Taniuti T. 1964)

Recept:

- Ustanovi se ravnotezno stanje
- Transfromisu se koordinate koristeci mali parametar ε – razvucene koordinate
- Perturbovane promenljive se razviju u red oko ravnoteznih vrednosti koristeci isti taj mali parametar
- Zameni se sve u pocetni sistem jednacina
- Sakupljaju se clanovi uz razlicite stepene ε

Analiza linearne disperzione jednacine

$$\omega^2 = \kappa^2 - 2\pi G \rho_0 |k|$$

- **Marginalna stabilnost** $k > k_1$ $k_1 = \rho_0' / \rho_0$ (vertikalna stabilnost diska)

  $\omega^2/\kappa^2 > 0$ STABILNO $|k| < k_2$ $k_2 = \frac{\kappa^2}{2\pi G \rho_0}$

  posmatranja: $k_1 \cong k_2$

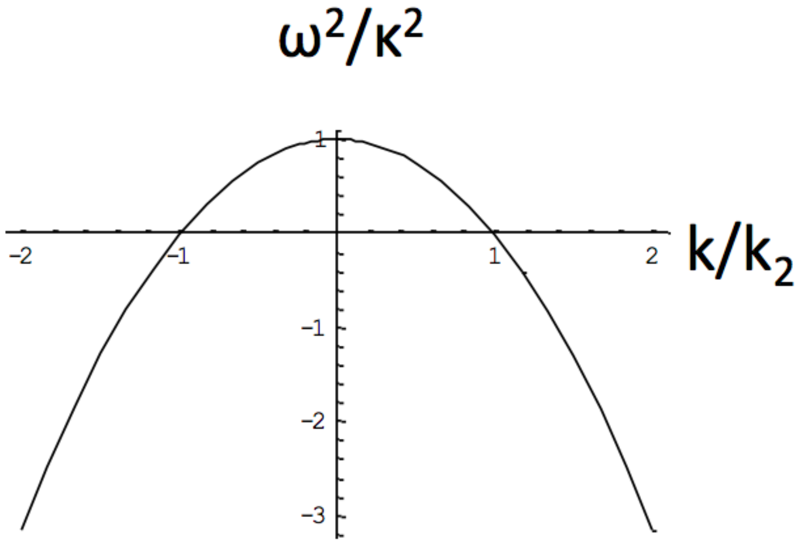


Fig. 1: dispersion relation

- Posebna transformacija koordinata jer grupna brzina tezi ∞ (ω --> 0) (Watanabe T. 1969)

$$\rho = \rho_0 + \sum_{\infty}^{n=1} \sum_{m=-\infty}^{\infty} \epsilon^n \rho^{(n,m)}(\xi, \eta) E,$$

$$\xi = \varepsilon(\tau - cr) \qquad \tau = t + \Omega^{-1}\varphi$$

$$\eta = \varepsilon^2 r$$

$$v_r = \sum_{\infty}^{n=1} \sum_{m=-\infty}^{\infty} \epsilon^n v^{(n,m)_r}(\xi, \eta) E,$$

$$v_\varphi = r\Omega + \sum_{\infty}^{n=1} \sum_{m=-\infty}^{\infty} \epsilon^n v_\varphi^{(n,m)}(\xi, \eta) E,$$

where $E = \mathrm{e}^{\mathrm{i}(kr - \omega\tau)}$.

$$\frac{\partial \phi}{\partial r} \simeq -r\Omega^2 + 2\pi G \epsilon \sum_{n=1}^{\infty} \varepsilon^n \sum_{l=1}^{\infty} \mathrm{Re}(2 i n_l^{(n)} \exp\{il[\omega t - m\theta + \lambda f(r)]\}),$$

$$\frac{\partial \phi}{\partial \theta} \simeq 0,$$

Sakupljajuci clanove uz ε¹, ε² i ε³ dobijamo odgovarajuce koeficijente

Kontrolni parametri: disperziona jednacina uz **ε¹**

$$\epsilon^1: m=0, \quad v_\varphi^{1,0} = a_1\rho^{1,0}, \quad a_1 = \frac{-i\pi G}{\Omega}, \quad v_r^{1,0} = 0;$$

$$m=1, \quad \boxed{\omega^2 = \kappa^2 - 2\pi G\rho_0 k,} \quad v_r^{1,1} = a_2\rho^{1,1}, \quad a_2 = \frac{-\omega}{k\rho_0},$$

$$v_\varphi^{1,1} = a_3\rho^{1,1}, \quad a_3 = \frac{-i\kappa^2}{2\Omega k\rho_0}.$$

$$\epsilon^2: m=0, \quad \rho^{1,0} = 0, \quad v_\varphi^{2,0} = a_4\rho^{2,0}, \quad a_4 = \frac{-i\pi G}{\Omega},$$

$$v_r^{2,0} = a_5\rho^{1,1}, \quad a_5 = \frac{2\omega}{k\rho_0^2};$$

i grupna brzina iz koeficijenata uz **ε²**

$$m=1, \quad \boxed{\frac{\partial\omega}{\partial k} = \frac{\pi G\rho_0}{\omega} = c,} \quad \rho^{2,1} = 0,$$

$$v_r^{2,1} = b_1\frac{\partial}{\partial\xi}\rho^{1,1}, \quad b_1 = \frac{\rho_0 c a_2 - 1}{i\rho_0 k},$$

$$v_\varphi^{2,1} = b_2\frac{\partial}{\partial\xi}\rho^{1,1}, \quad b_2 = \frac{a_3 - \frac{\kappa^2}{2\Omega}b_1}{i\omega};$$

$$m=2, \quad v_r^{2,2} = b_3(\rho^{1,1})^2, \quad b_3 = \frac{\frac{1}{2}ika_2^2 + \frac{1}{2}\frac{k\Omega}{\omega}a_2a_3 + \frac{i\pi Gk}{\omega}a_2}{i\omega + \frac{\kappa^2}{4i\omega} - \frac{i\pi Gk\rho_0}{\omega}},$$

$$\rho^{2,2} = b_4(\rho^{1,1})^2, \quad b_4 = \frac{k}{\omega}a_2 + \frac{\rho_0 k}{\omega},$$

$$v_\varphi^{2,2} = b_5(\rho^{1,1})^2, \quad b_5 = \frac{1}{2}\frac{k}{\omega}a_2a_3 - \frac{\kappa^2}{4i\omega\Omega}b_3;$$

$$\epsilon^3: m=0, \rho^{2,0}=0; \quad m=1, \frac{\omega}{2\pi G}b_2 + \frac{2\Omega}{2i\pi G}b_3 = b_1;$$

Zamenjujuci parametre u razvoju **ε³**

dobija se Nelinearna Sredingerova jednacina

$$\boxed{i\frac{\partial}{\partial\eta}\rho^{1,1} + P\frac{\partial^2}{\partial\xi^2}\rho^{1,1} + Q\left|\rho^{1,1}\right|^2\rho^{1,1} = 0.}$$

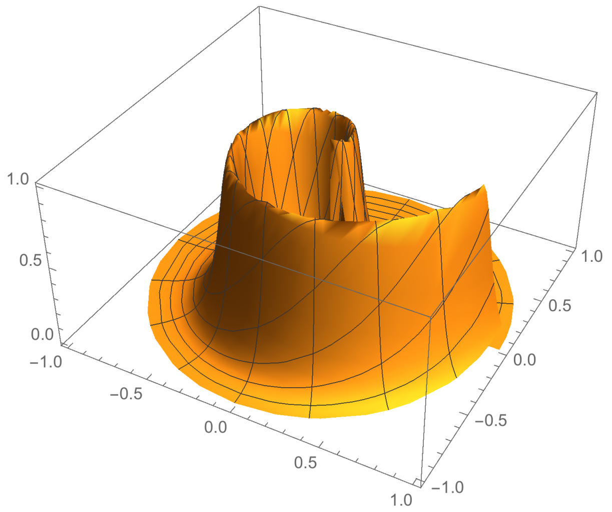
Resenje

$$\rho^{1,1}(\xi,\eta) = \rho_a \frac{e^{i\psi}}{ch(\sqrt{\frac{Q}{P}}\rho_a(\xi - P\eta))}.$$

svetli soliton – povecanje gustine

fazno pomereni gustina i potencijal

$$P = k_2/\kappa = \kappa/2\pi G\rho_0 = 1/V_g \qquad Q = \kappa^3/(k_2\rho_0^2)$$



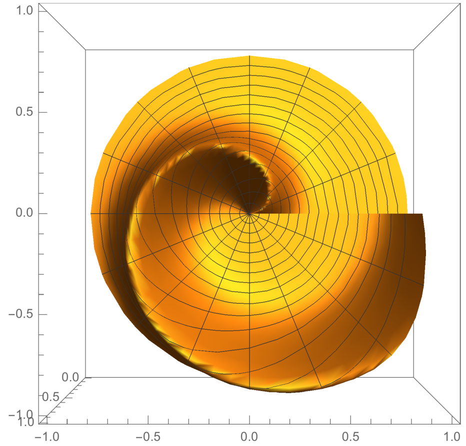
Originalne koordinate – jednodimenzioni zakrivljeni talas

Spiralni oblik

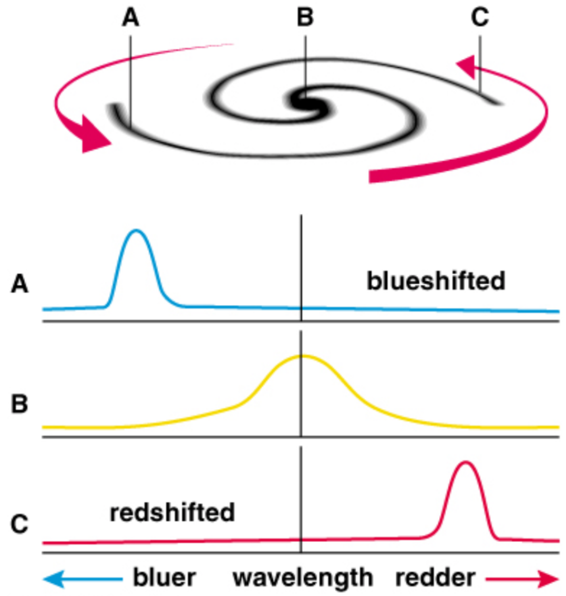
Osobine: konstantna grupna brzina i konstantna amplituda

Balans disperzije i nelinearnih efekata

Disperzija



Izraz za rotacionu brzinu zvezdane komponente spiralnih galaksija

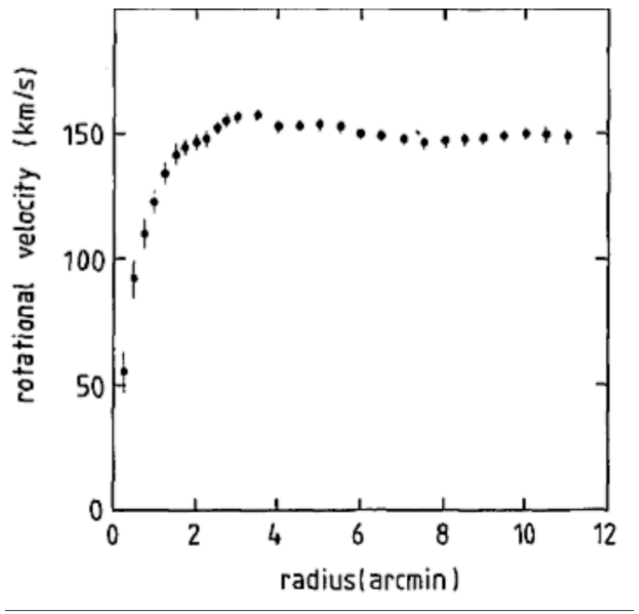


Doppler effect

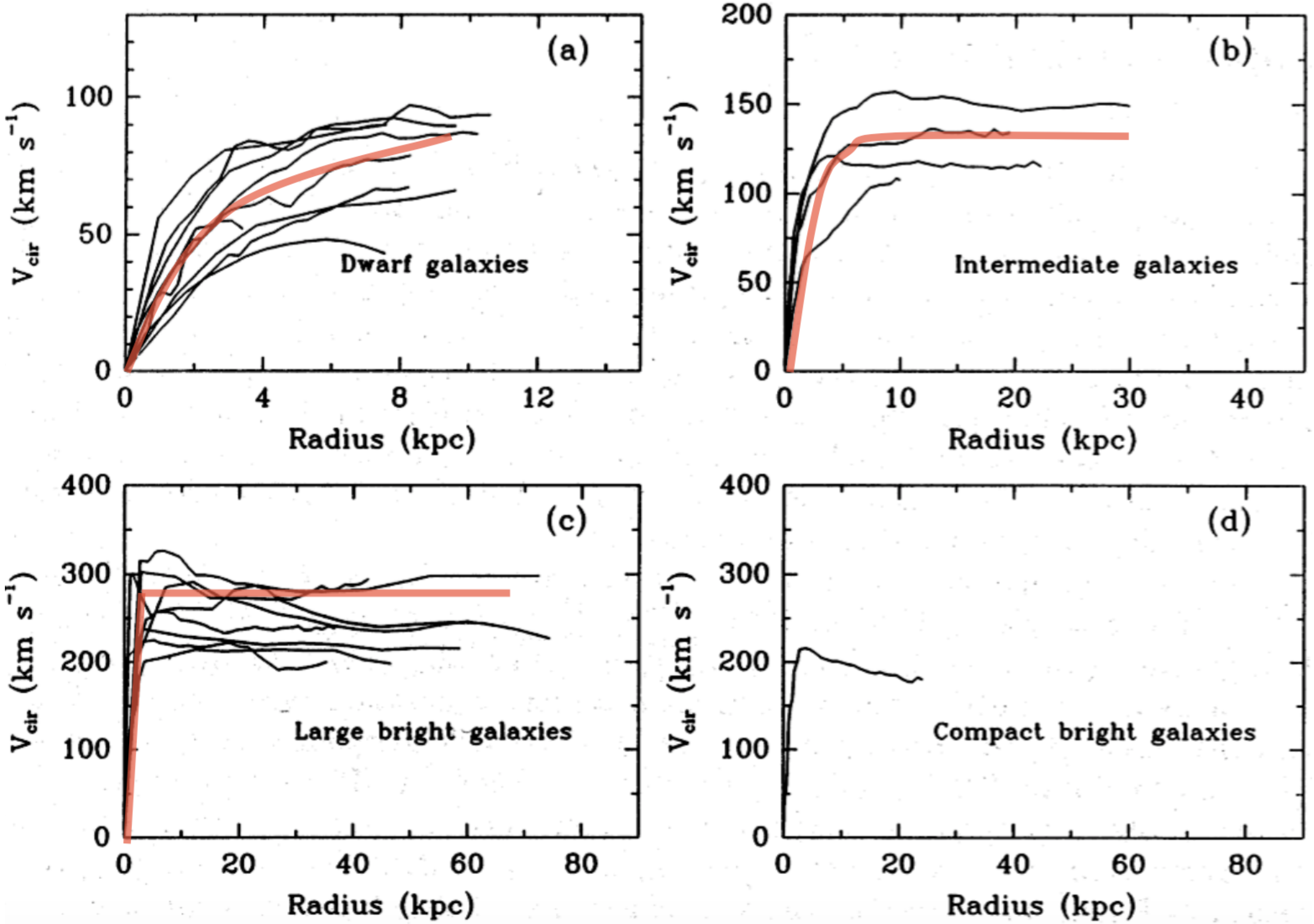
$$v_r \equiv \frac{\Delta\lambda}{\lambda_{HI}} c$$

radijalna brzina iz Doplerovog efekta

rotaciona brzina za razlicite radijuse



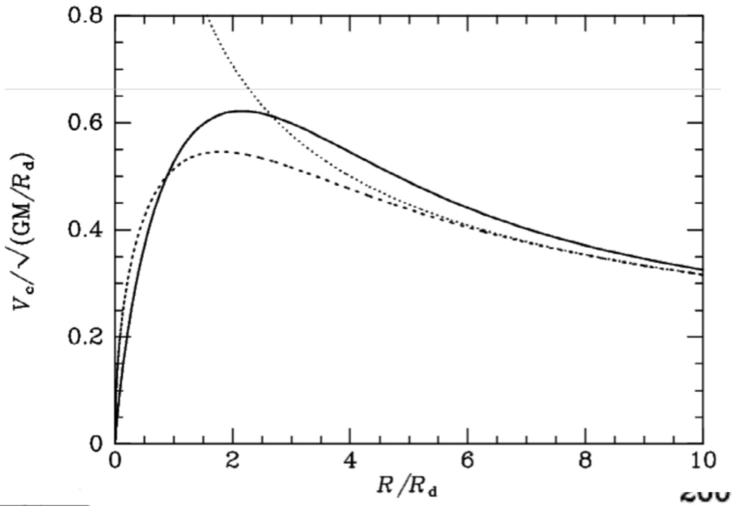
NGC 3198



Teorijski model

- Rotaciona brzina

$$v_c(r) = \sqrt{\frac{GM}{r}} \propto r^{-1/2}$$

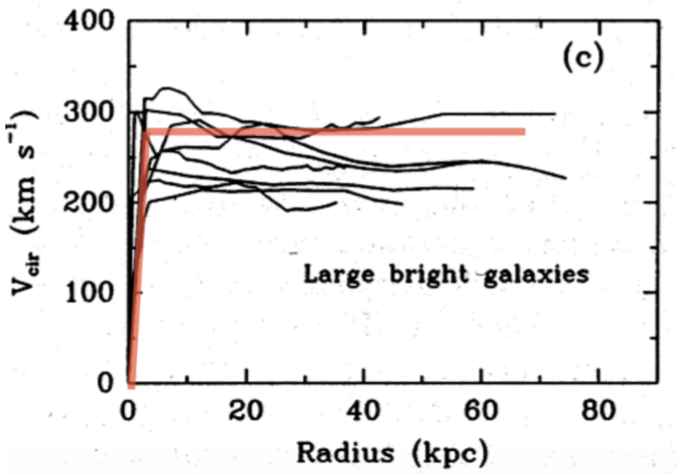


sva masa u tacki – tackasta linija

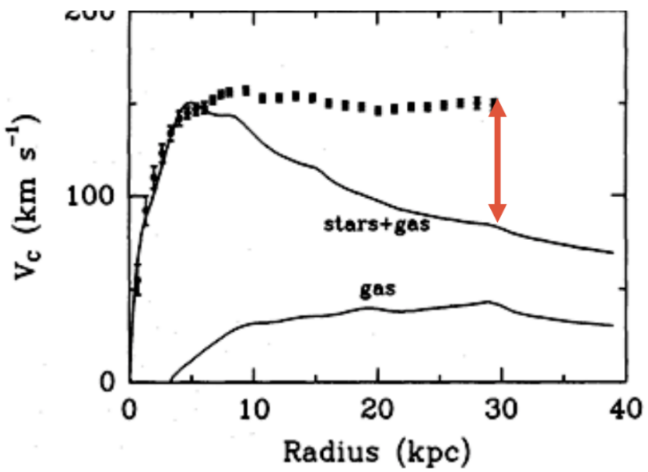
sferno telo iste mase – isprekidana

eksponencijalna raspodela mase – puna

(Binney and Tremaine, 1987)



observations



theoretical model

- most of spiral galaxies have **flat** rotation curve
- Lack of mass – introduced invisible (undetectable) mass distributed in spherical halo

- Umesto aproksimacije exponencijalnog diska za povrsinsku gustinu koristimo resenje NSJ

$$V^2(r) = r\frac{\partial \phi}{\partial r}.$$

$$\frac{\partial \phi}{\partial r} = r\Omega^2 + \sum_{\infty}^{n=1} \sum_{m=-\infty}^{\infty} 2\pi G \epsilon^n \Re(\rho^{(n,m)}(\xi,\eta) e^{i(kr-\omega\tau)}),$$

$$\rho^{1,1}(\xi,\eta) = \rho_a \frac{e^{i\psi}}{ch(\sqrt{\frac{Q}{P}\rho_a}(\xi - P\eta))}.$$

$$V(r) = \sqrt{\Omega^2 r^2 + \frac{ar}{\cosh b(T - cr)}}.$$

- Koeficijenti a, b i c se odredjuju tako sto se vratimo na osnovne polarne koordinate, V_g se mnozi sa $2\pi G\rho_0/\kappa$, radijus je izrazen u km, $T = 1 = (t + \varphi/\Omega)$ je izrazeno u s, a φ je polarni ugao

$$a = 2\pi G\rho_0\rho_a [km/s^2]$$

Ω^2 [1/s^2] ali brojna vrednost u samom izrazu odredjuje se tako da r [km]

$$b = \kappa\rho_a [1/s]$$

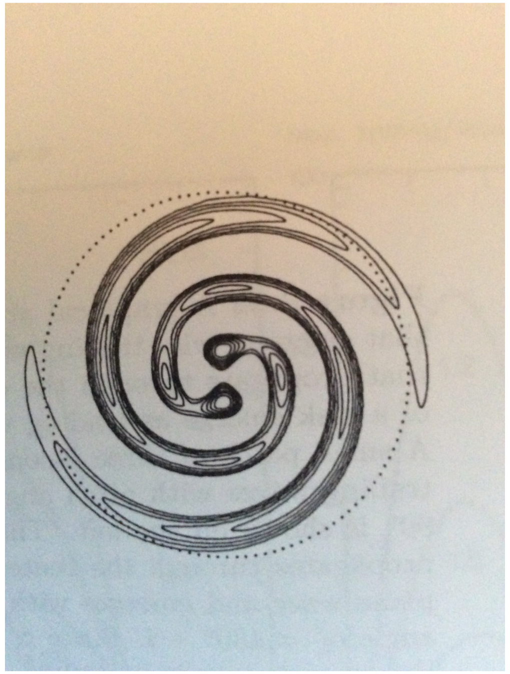
$$c = 1/V_g [s/km]$$

$$V_g = \pi G\rho_0/\kappa$$

relative wave amplitude ρ_a (density enhancement along the spiral) $(\rho_a \sim 0.3)$ (3-5)% normirano sa ρ_0

$V_g = \pi G \rho_0/\kappa \sim 200 km/s$ za tipicne vrednosti $(\kappa \sim 10^{-15} 1/s,\ \rho_0 \sim (4-6)\times 10^{-2} g/cm^2 = (200-300) M_\odot/pc^2)$

The group velocity is tangent on the spiral at given r, while rotational velocity is tangent on the circle at given r, so that $V_g = V\cos\alpha$ where α is angle between the spiral and circle at given r. This angle is very small and $\cos\alpha \simeq 1$. Thus, this wave velocity coincides with the rotational velocity of particles as long as the soliton wave exists.



Domen u kome vazi nase resenje odredjen je kriticnim talasnim vektorima k_1 i k_2

$k = 2\pi/\lambda$, $\quad k_1 = max[1/r, \rho_0'/\rho_0]$ and $k_2 = \kappa^2/2\pi G\rho_0$

$2\pi G\rho_0/\kappa^2 < r < \rho_0/\rho_0'$ $\quad$ od $r \sim 1\text{kpc}$ do $r \to \infty$

Ugaona brzina i differencijalna rotacija:

u centralnim delovima galaksije $\quad \kappa \simeq 2\Omega$

u spoljnim delovima diska $\quad \kappa \simeq \Omega$

$\Omega \leqslant \kappa \leqslant 2\Omega$ ili $\sqrt{2}\Omega$ od (2-3)kpc do kraja diska

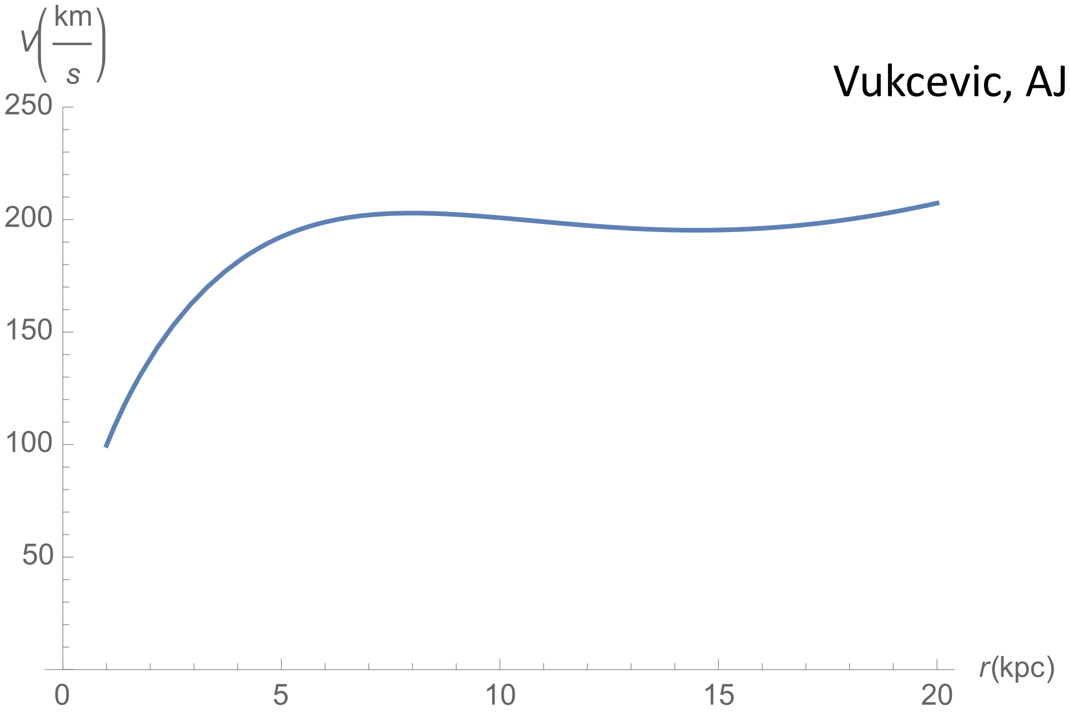
(Binney and Tremaine, 1987)

Primena relacije rotacione brzine na Mlecni put i nekoliko tipicnih i netipicnih spiralnih galaksija

Povrsinska gustina $\rho_0 = 0.045 g/cm^2 = 200 M_\odot / pc^2$

Ugaona brzina $\Omega \sim 30 \times 10^{-16} 1/s$ $V_g = 200 km/s$ (Luna et al. 2006)

$\Omega^2 = 90\ [km/(s\ kpc)]^2$ $a = 6 \times 10^3$ $b = 4.5 \times 10^{-16}$ $c = 4 \times 10^{14}$



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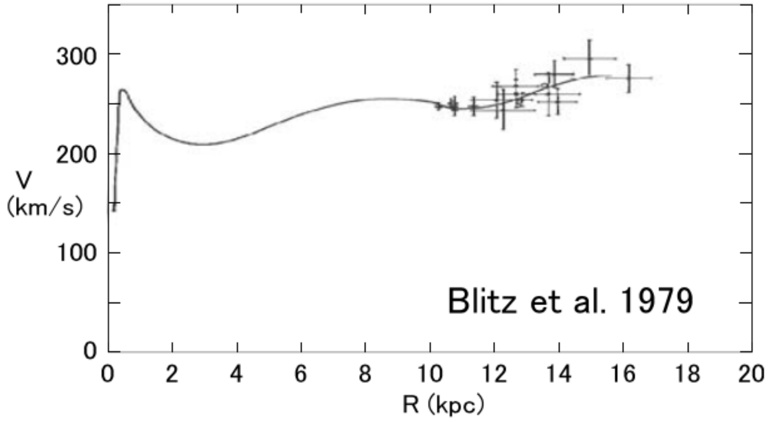
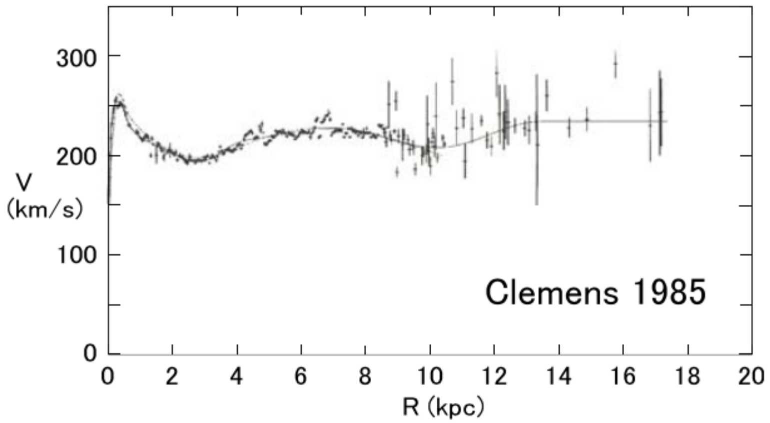
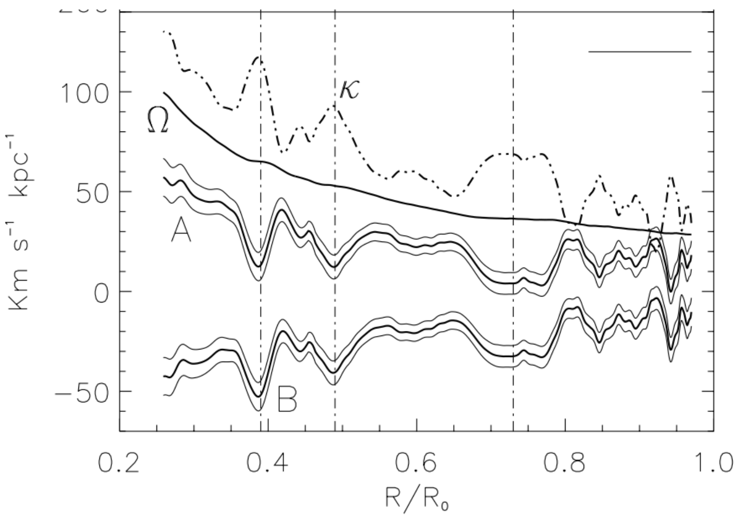




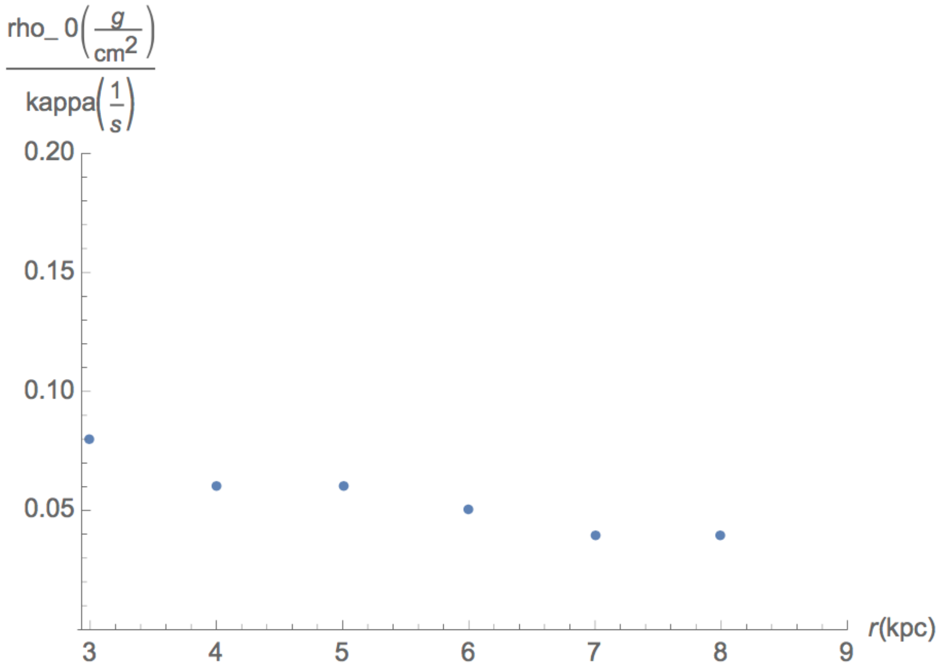
$\Omega=\Omega(r)$ $\rho_0=\rho_0(r)$ $\kappa=\kappa(r)$

$a, b, V_g \sim \rho_0/\kappa$



Galactic Radius[a] (R/R_0)	Circular Velocity[b] (km s^{-1})	Σ_{gas}[c] ($M_\odot$ pc^{-2})
0.325	212.4 ± 2	3.57 ± 0.03
0.375	209.6 ± 2	3.93 ± 0.03
0.425	214.2 ± 2	2.33 ± 0.03
0.475	215.7 ± 2	3.06 ± 0.02
0.525	223.6 ± 2	4.12 ± 0.02
0.575	219.3 ± 2	1.94 ± 0.02
0.625	218.0 ± 2	1.66 ± 0.02
0.675	214.5 ± 2	2.19 ± 0.01
0.725	225.3 ± 2	3.92 ± 0.01
0.775	236.7 ± 2	2.40 ± 0.01
0.825	234.3 ± 2	1.68 ± 0.01
0.875	236.1 ± 2	0.84 ± 0.01
0.925	233.1 ± 2	0.41 ± 0.01
0.975	233.6 ± 2	0.33 ± 0.01

(Luna et al. 2006)



$\rho_0/\kappa \sim$ const!

Uticaj $\Omega=\Omega(r)$ i $\rho_0=\rho_0(r)$ na oblik rotacione krive

FICH, BLITZ, AND STARK

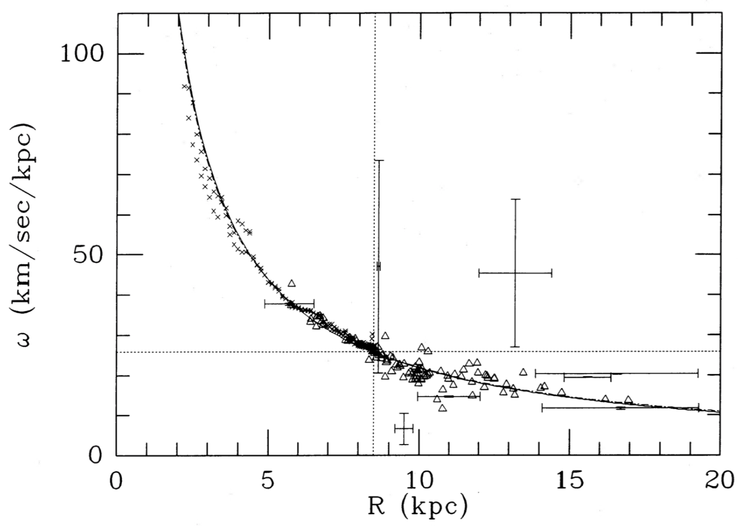


FIG. 2a

FIG. 2.—The data points used for the rotation curve determinations (*crosses* from H I rangent point data, *triangles* from CO data). Error bars are shown for a few outlying H II regions. Error bars for H II regions near the center of the distribution and for the H I points are in general smaller than the symbols used to plot the positions. (*a*) ω vs. R plot. (*b*) Θ vs. R plot. We show error bars for a few of the most uncertain CO data points. The "best-fit" linear (*solid line*) and power law (*dashed line*) rotation curves for the IAU standard values of $R_0 = 8.5$ kpc and $\Theta_0 = 220$ km s^{-1} are shown.

Figures 2*a* and 2*b* show the "best fit" rotation curves for the new IAU standard values for R_0 and Θ_0 of 8.5 kpc and 220 km s^{-1}. The linear fit shown is the function

$$\frac{\omega}{\omega_0} = 1.00746\left(\frac{R_0}{R}\right) - 0.017112 \qquad (21)$$

and the power-law fit shown is

$$\frac{\omega}{\omega_0} = 0.49627\left(\frac{R_0}{R}\right)^{0.99579} + 0.49632\left(\frac{R_0}{R}\right). \qquad (22)$$

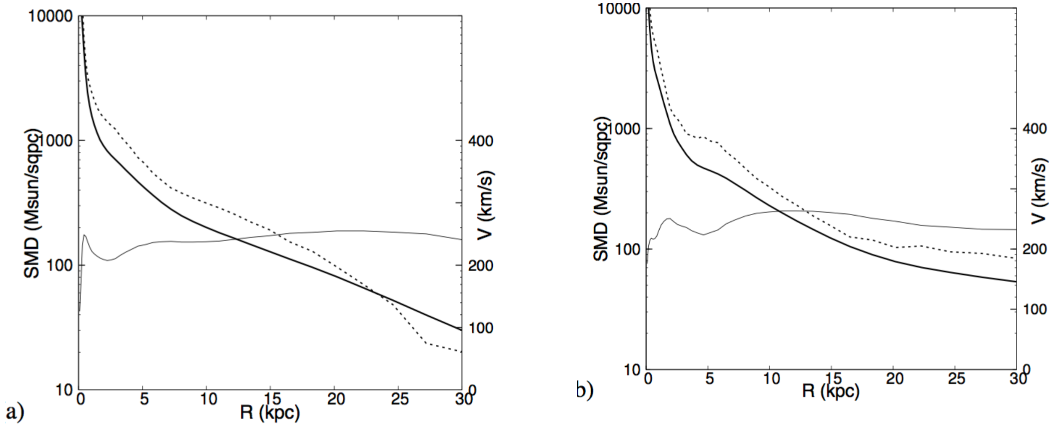


Fig. 2. (a) Radial profiles of SMD-F (thick line), SMD-S (dashed line) and RC (thin line) for the Milky Way, and (b) M31.

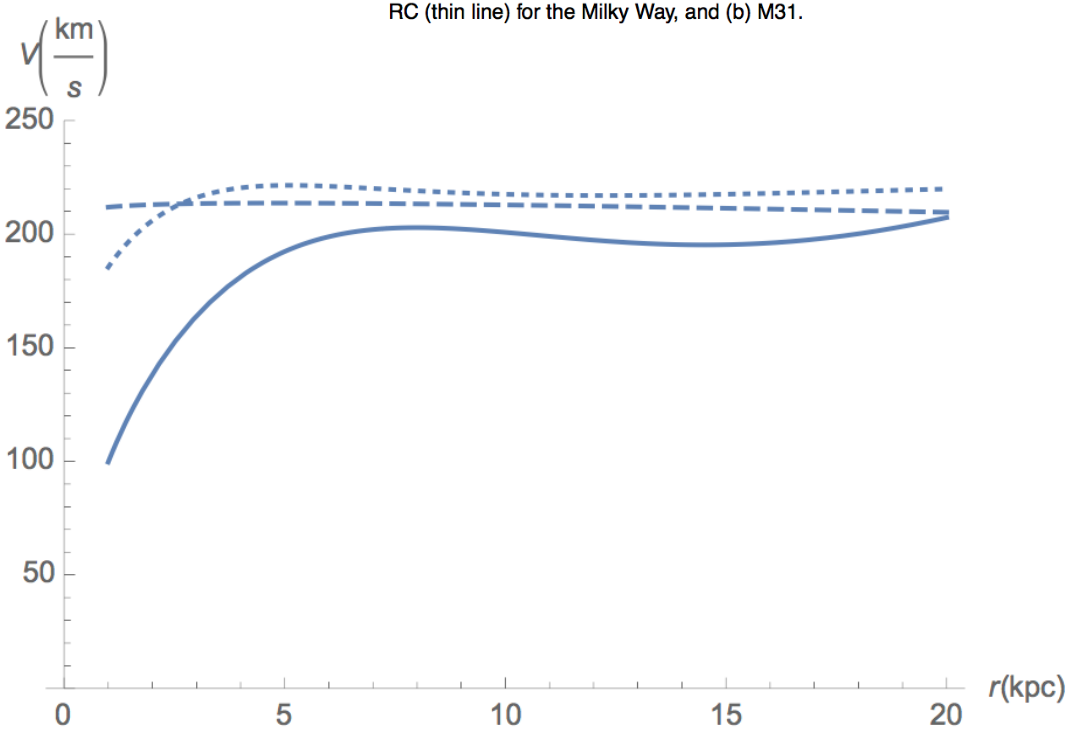


Figure 1. Rotational velocity curve for $\Omega = const.$ presented by solid line. Dotted line is rotational velocity curve for power-law fit given by Eq. (22) in Fich et al. (Fich et al. 1989), namely $\Omega(r) \sim r^{-0.9}$, while dashed line is result obtained from Eq. (6) taking all variables r dependent ($\rho(r) \sim r^{(-0.7)}$; power-law for SMD is approximated according to result obtained by Sofue for Milky Way and M31 (Sofue 2018), and $\Omega(r) \sim r^{-0.9}$).

Efekti debljine diska:

- najnestabilnija struktura n→0
- precenili smo intenzitet rotacione brzine

$$A\nabla_{\perp}^{2}\bar{\phi} + B\bar{\phi} = \bar{\rho},$$

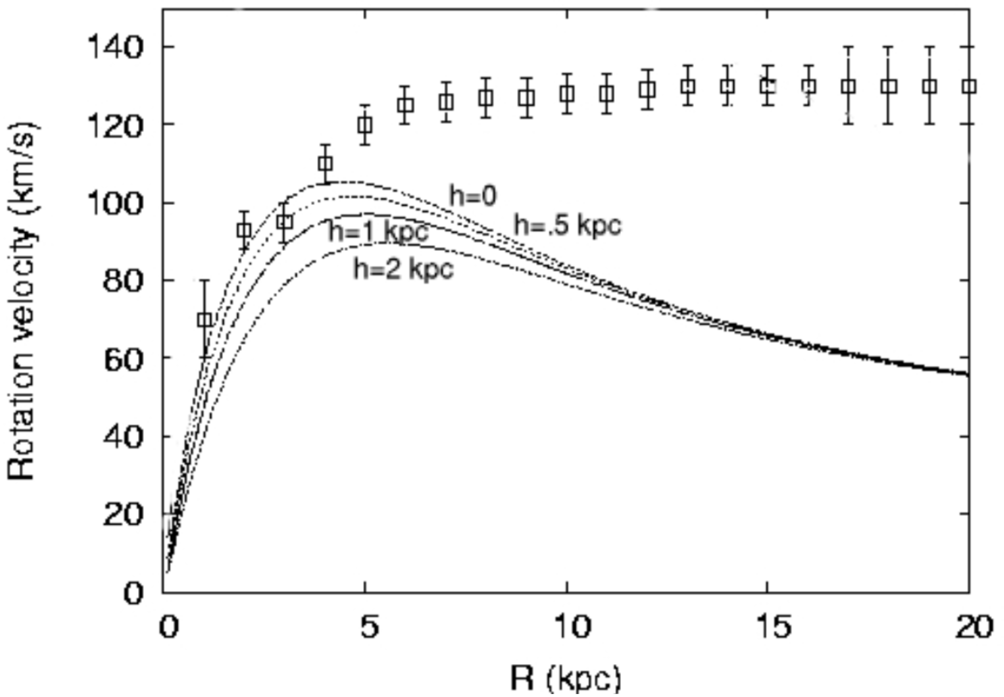
$$\mathrm{i}\frac{\partial}{\partial\eta}\rho^{1,1} + W\frac{\partial^{2}}{\partial\xi^{2}}\rho^{1,1} + Z\left|\rho^{1,1}\right|^{2}\rho^{1,1} = 0,$$

$$W = -\frac{k_2}{n\kappa^2} \qquad Z = -\frac{3}{2}\frac{n\kappa^2}{k_2\rho_0^2}$$

$$(\omega - m\Omega)^2 = \kappa^2 - \frac{4\pi G\rho_0 m\hat{k}^2}{1+\hat{k}^2},$$

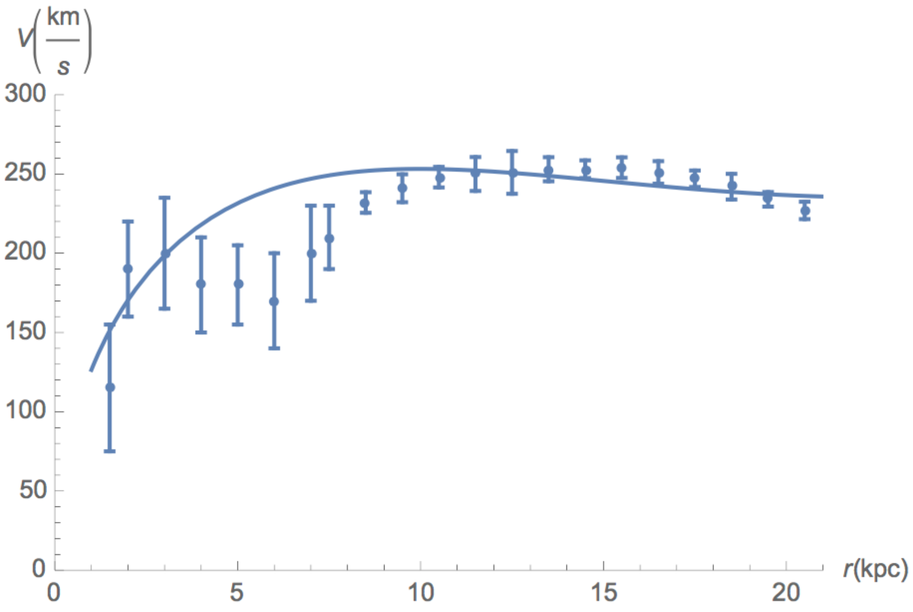
where $\hat{k}^2 = \frac{k^2}{n}$, and $m = 1/A$, $n = 1/B$.

(Vukcevic, MNRAS, 2014)



Exponential disc fit to the RC of NGC 2403

- Kompenzuje se gasnom komponentom



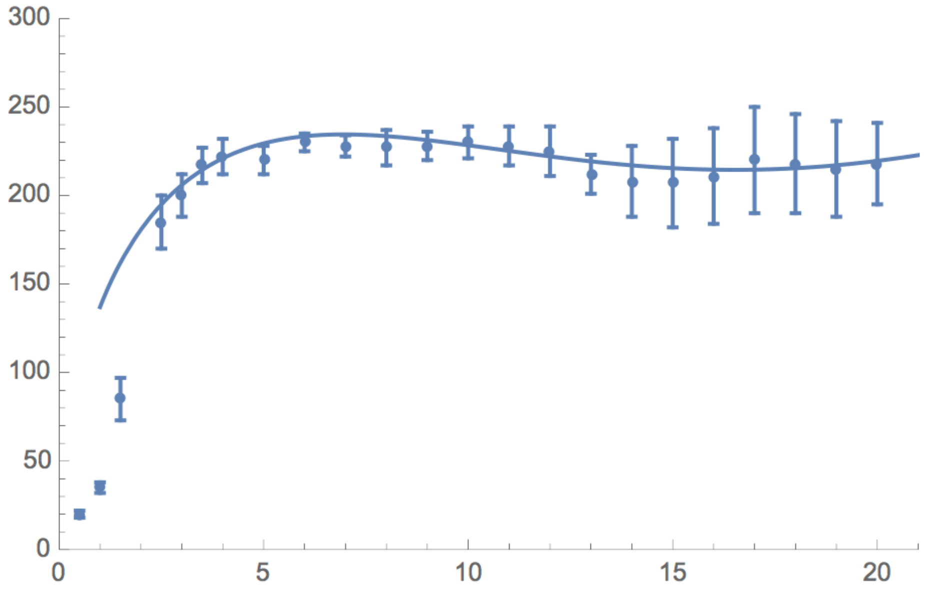


Fig. 1: Rotaciona kriva M31. Tacke su posmatranja (Corbelli et al. 2010); puna linija iz jednacine za rotacionu brzinu za sledece parametre: $\Omega^2=50$, $a=7x10^3$, $b=4x10^{-16}$, $c=3.7x10^{14}$, dobijene za vrednosti iz posmatranja, $\Omega=20x10^{-16}$1/s, $\rho_0=4.9x10^{-2}$ g/cm^2, $V_g=230$km/s

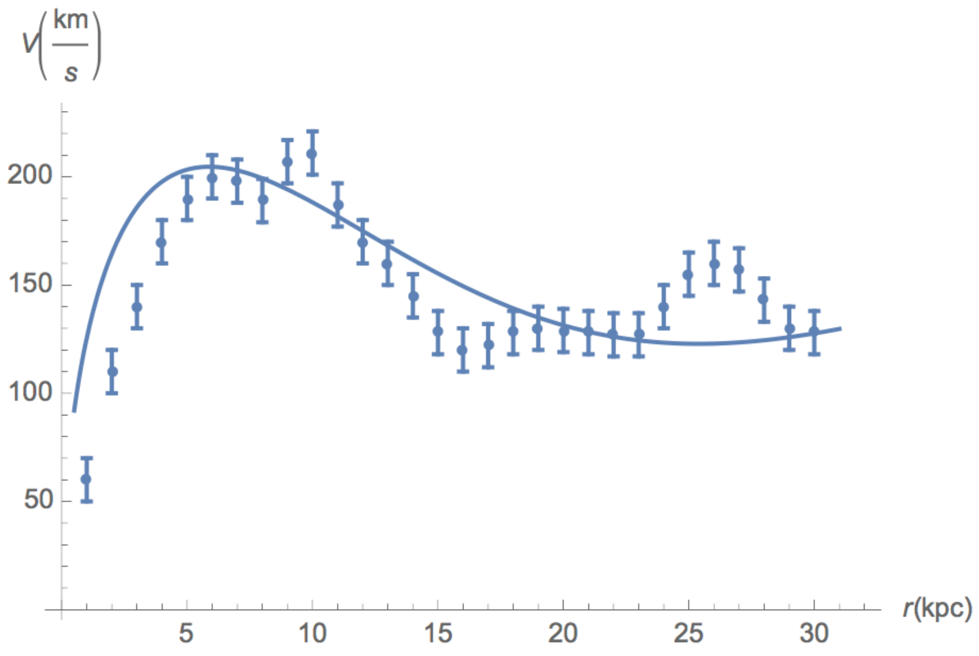
Fig. 2: Rotaciona kriva tipicne galaksije NGC 3521. Tacke su posmatranja (de Blok et al. 2008); puna linija iz jednacine za rotacionu brzinu za sledece parametre: $\Omega^2=80$, $a=6.5x10^3$, $b=4.2x10^{-16}$, $c=3.8x10^{14}$, dobijene za vrednosti iz posmatranja $\Omega=28x10^{-16}$, $\rho_0=4.3x10^{-2}$g/cm^2, $V_g=220$km/s

Netipicna spiralna galaksija – ceka objasnjenje

NGC 157

$\Omega=7x10^{-16}1/s$, $\rho_0=2.4x10^{-2}$ g/cm^2, $V_g=180km/s$

$\Omega^2=15$, $a=6x10^3$, $b=3.8x10^{-16}$, $c=3.6x10^{14}$



N-body simulacije

- 2D/3D gravitacione N-body simulacije; GADGET code (Springel, MNRAS, 2005).
- **Direktni pristup**: input spiralna raspodela povrsinske masene gustine; eksplicitno resenje nelinearne Schrödinger-ove jednacine. Halo sa tamnom materijom nije uracunat.

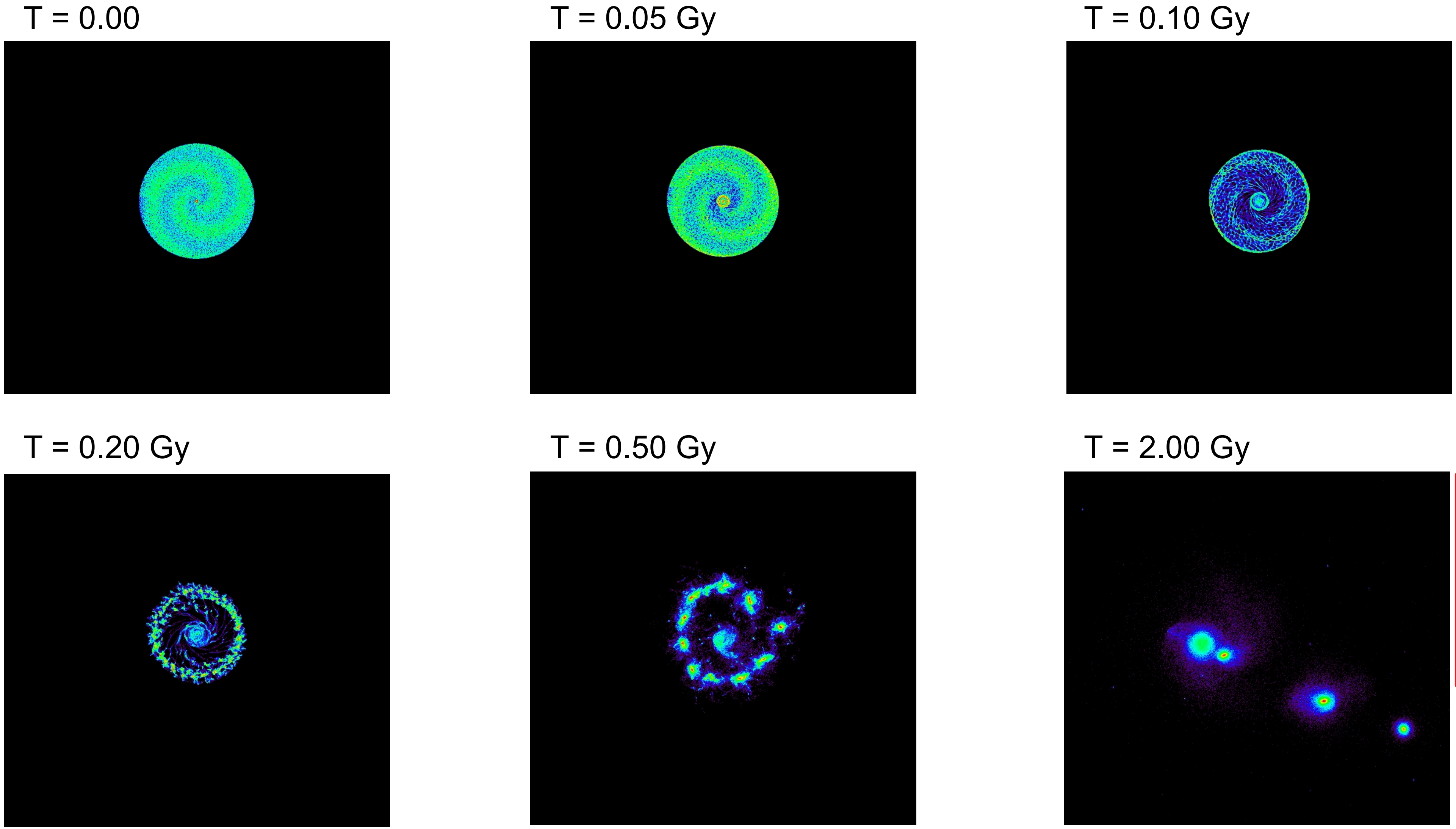


FIG. 1 The simulation run with initial density distribution set as **solution to non-linear Schrödinger equation**. The initial conditions:
M ~ 10^11 M☼
N ~ 10^7 particles,
V ~ 200 km/s,
R ~ 30 kpc.
The plots' size is 100x100 kpc.

The diffusing mass (R > 50kpc):

less than 25%

• **Evolutivni pristup**: input je nelinearno vrtlozno resenje za povrsinsku gustinu u baldzu koje formira spiralnu strukturu u disku.Ponovo, halo sa tamnom materijom nije uracunat.
• Solitonska struktura (spiralne grane) ostaje stabilna (reda velicine 2 Gy) za disk+bulge konfiguraciju na R ~ 40 kpc.

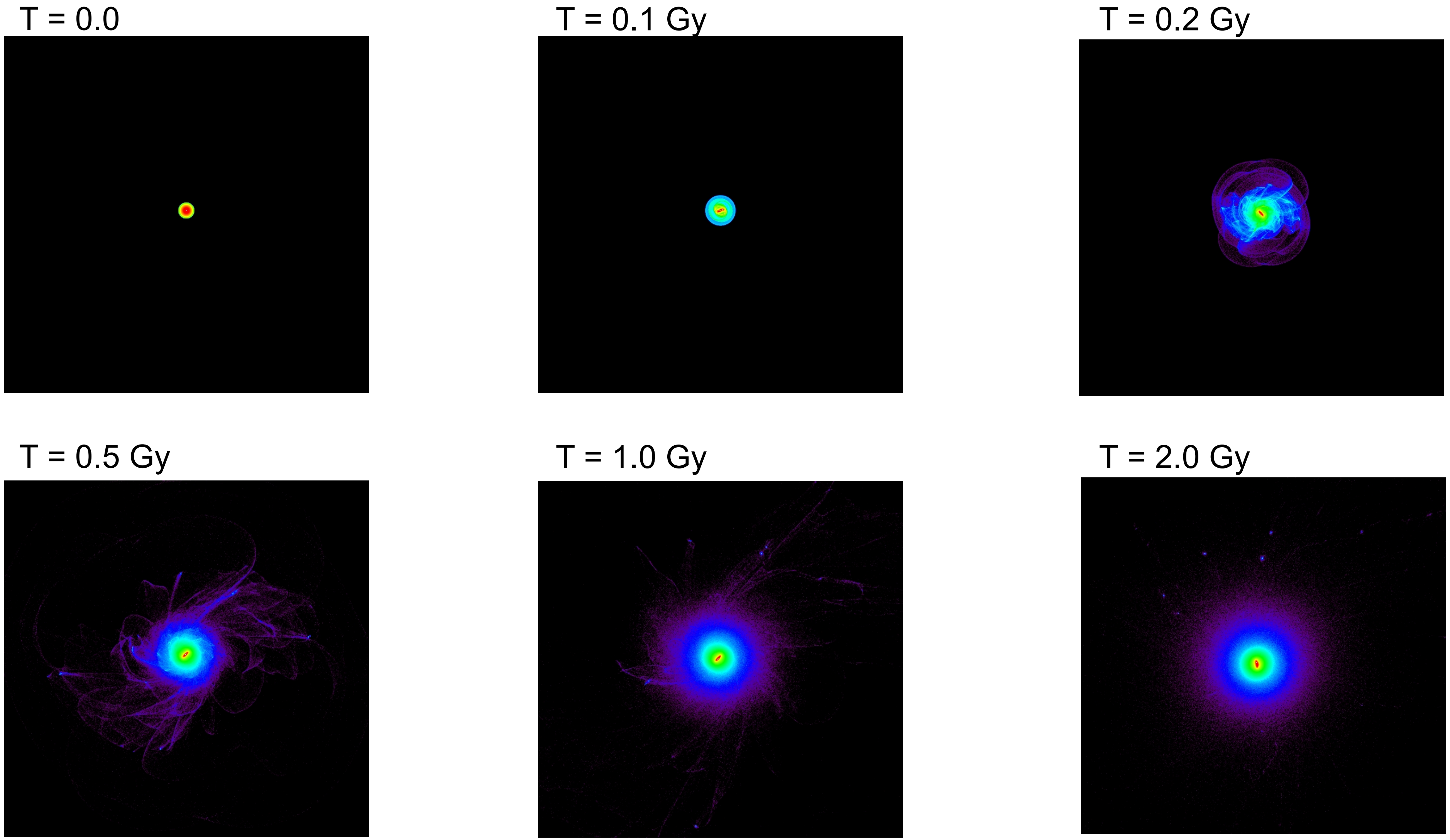


FIG. 2 The simulation of **evolving initial mass**, set as non-linear vortex solution. The initial conditions:
M ~ 10^11 M☼
N ~ 10^7 particles,
R ~ 5 kpc,
V ~ 200 km/s.
The plots’ size is 100x100 kpc.

The diffusing mass (R > 50kpc):

less than 5%

Zakljucak

- Postojanje nelinearne jednacine implicira konstantnu grupnu brzinu i konstantnu amplitudu talasa
- Pravac i smer te grupne brzine su direktna posledica uslova marginalne stabilnosti diska koja se dobija iz posmatranja --- vec od 3 kpc grupna brzina se poklapa sa rotacionom brzinom --- cirkularno osnosimetricno kretanje
- Ugaona brzina i raspodela mase u disku nisu medjusobno nezavisne promenljive
- Simulacije ukazuju na stabilnu spiralnu struktru reda velicine 2Gy bez potrebe za haloom i tamnom materijom
- Nelinearni efekti definitivno zahtevaju preispitivanje kolicine tamne materije potrebne u dinamici spiralnih galaksija

- Zanemarivanje efekata ciji nam doprinos izgleda mali i zanemarljiv nije uvek opravdano
- Pre nego sto u pomoc pozovemo fenomen koji nema detektabilnu osobinu, mozda je bolje da preispitamo pretpostavke koje smo napravili
- Ali je dobro imati siru sliku i ostaviti mogucnost da tamo negde, postoji nesto, sto jos uvek ne mozemo da detektujemo direktno

Hvala na paznji!