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Modified equipartition calculation for supernova remnants

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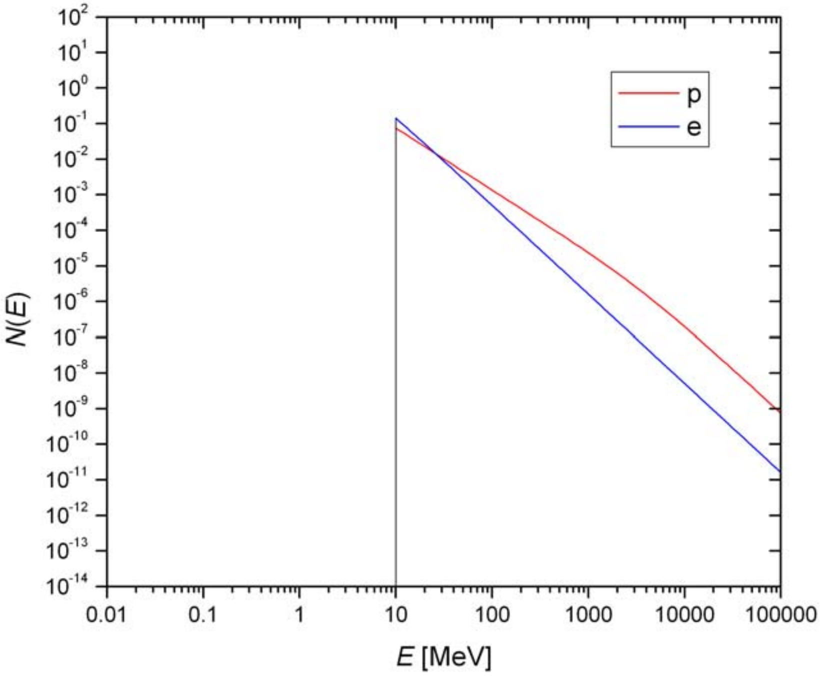
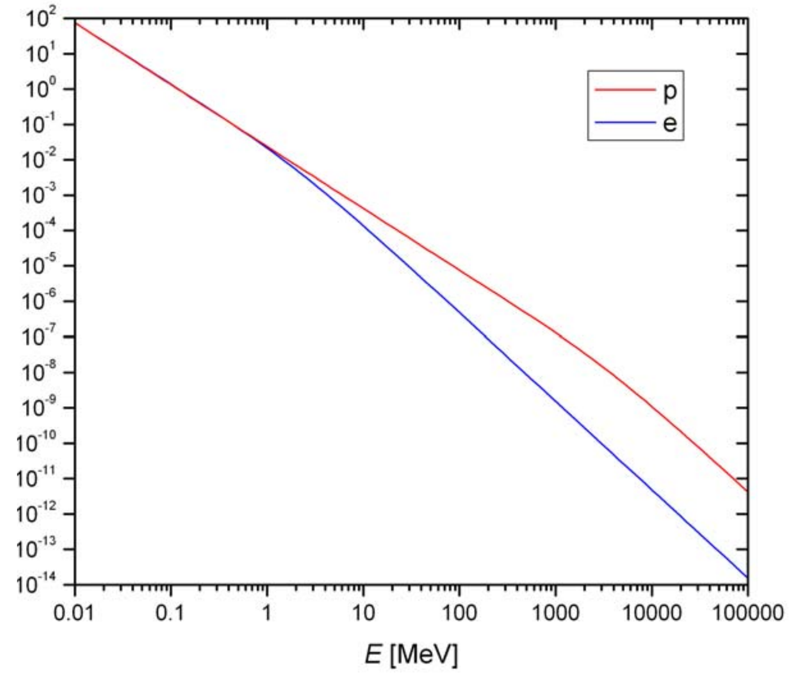
Equipartition calculation

- The equipartition or minimum-energy calculation
- determination of the magnetic field strength and minimal energy contained in the magnetic field and cosmic ray particles
- directly from radio-continuum observations – synchrotron emission
- Pacholczyk (1970) – integration of the cosmic ray energy spectrum over frequencies – “classical equipartition”
- Revised equipartition – Beck & Krause (2005) – integration of the cosmic ray energy spectrum over energies

- The energy spectrum





Simple approach

- Bell's (1978) injection $E_{\text{inj}} \approx 4\ ½\ m_p v_s^2$

- Shock velocity of an SNR is low enough (v_s << 7000 km/s – older SNRs) =>

  $E_{\text{inj}} << m_e c^2$ ($p^{\text{inj}}_e << m_e c$)
- Particles are injected into the acceleration process all with the same injection energy

- Plasma is fully ionized and globally electroneutral

Derivation

- Assuming power law momentum distribution ($N=kp^{-\gamma}$), the energy density of one cosmic ray “ingredient” is:

$$
\begin{aligned}
\epsilon &= \int_{p_{\rm inj}}^{p_\infty} 4\pi k p^{-\gamma}(\sqrt{p^2c^2+m^2c^4}-mc^2)\mathrm{d}p \\
&\approx \int_0^\infty 4\pi k p^{-\gamma}(\sqrt{p^2c^2+m^2c^4}-mc^2)\mathrm{d}p \\
&= 4\pi kc(mc)^{2-\gamma}\int_0^\infty x^{-\gamma}(\sqrt{x^2+1}-1)\mathrm{d}x, \quad x=\frac{p}{mc} \\
&= K(mc^2)^{2-\gamma}\frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}.
\end{aligned}
$$

- K is from $N=KE^{-\gamma}$; convergent solutions only for $2 < \gamma < 3$ (the spectral indices of SNRs: $0.5 < \alpha < 1$)

- Total CR energy density (for all particles: electrons, protons, heavier ions):

$$\begin{aligned}
\epsilon_{\rm CR} &= \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}\left(K_e(m_ec^2)^{2-\gamma}+\sum_i K_i(m_ic^2)^{2-\gamma}\right)\\
&= \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}\left(K_e(m_ec^2)^{2-\gamma}+K_p(m_pc^2)^{2-\gamma}\sum_i \frac{n_i}{n_p}\left(\frac{m_i}{m_p}\right)^{(3-\gamma)/2}\right)\\
&= \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}K_e(m_ec^2)^{2-\gamma}\left(1+\frac{n}{n_e}\left(\frac{m_p}{m_e}\right)^{(3-\gamma)/2}\sum_i \frac{n_i}{n}\left(\frac{m_i}{m_p}\right)^{(3-\gamma)/2}\right)\\
&= K_e(m_ec^2)^{2-\gamma}\frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}(1+\kappa), \qquad (2)
\end{aligned}$$

$$\kappa = \left(\frac{m_p}{m_e}\right)^{(3-\gamma)/2} \frac{\sum_i A_i^{(3-\gamma)/2} \nu_i}{\sum_i Z_i \nu_i}$$

where κ is the energy ratio, ν_i are the ion abundances, A_i and Z_i are masses and charge numbers of elements; we neglected energy losses.

- The synchrotron emissivity:

$$\varepsilon_\nu = c_5 K_e (B \sin\Theta)^{(\gamma+1)/2} \left(\frac{\nu}{2c_1}\right)^{(1-\gamma)/2}$$

- The flux density:

$$S_\nu = \frac{L_\nu}{4\pi d^2} = \frac{\frac{4\pi}{3} R^3 f \mathcal{E}_\nu}{4\pi d^2} = \frac{4\pi}{3} \varepsilon_\nu f \theta^3 d$$

- The isotropic distribution for the orientation of pitch angles (radial magnetic field) (Longair 1994) :

$$\frac{1}{2}\int_0^\pi (\sin\Theta)^{(\gamma+3)/2} \mathrm{d}\Theta = \frac{\sqrt{\pi}}{2} \frac{\Gamma(\frac{\gamma+5}{4})}{\Gamma(\frac{\gamma+7}{4})}$$

- For the total energy we have:

$$E = \frac{4\pi}{3} R^3 f(\epsilon_{\rm CR} + \epsilon_B), \quad \epsilon_B = \frac{1}{8\pi} B^2,$$

$$E = \frac{4\pi}{3} R^3 f\left(K_e (m_e c^2)^{2-\gamma} \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}(1+\kappa) + \frac{1}{8\pi} B^2 \right)$$

- Looking for the minimum energy with respect to *B*, d*E*/d*B*=0:

$$\frac{\mathrm{d}K_e}{\mathrm{d}B} (m_e c^2)^{2-\gamma} \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}(1+\kappa) + \frac{1}{4\pi} B = 0$$

- Using eqs. for synchrotron emissivity, flux density and angular distribution of magnetic field:

$$\frac{\mathrm{d}K_e}{\mathrm{d}B} = -\frac{3}{4\pi}\frac{S_\nu}{f\theta^3 d}\frac{1}{c_5}\left(\frac{\nu}{2c_1}\right)^{-(1-\gamma)/2}\frac{(\gamma+1)\Gamma(\frac{\gamma+7}{4})}{\sqrt{\pi}\Gamma(\frac{\gamma+5}{4})}B^{-(\gamma+3)/2}$$

- Finally:

$$B = \Big(\frac{3}{2\pi}\frac{(\gamma+1)\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})\Gamma(\frac{\gamma+7}{4})}{(\gamma-1)\Gamma(\frac{\gamma+5}{4})}\frac{S_\nu}{f d\theta^3}\cdot$$

$$\cdot(m_e c^2)^{2-\gamma}\frac{(2c_1)^{(1-\gamma)/2}}{c_5}(1+\kappa)\nu^{(\gamma-1)/2}\Big)^{2/(\gamma+5)}$$

- For easier calculation:

$$B\ [\mathrm{Ga}] \approx \left(6.286 \cdot 10^{(9\gamma-79)/2} \frac{\gamma+1}{\gamma-1} \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})\Gamma(\frac{\gamma+7}{4})}{\Gamma(\frac{\gamma+5}{4})} (m_e c^2)^{2-\gamma} \cdot \right. \quad (12)$$

$$\left. \cdot \quad \frac{(2c_1)^{(1-\gamma)/2}}{c_5} (1+\kappa) \frac{S_\nu [\mathrm{Jy}]}{f\ d[\mathrm{kpc}]\ \theta[\mathrm{arcmin}]^3} \nu[\mathrm{GHz}]^{(\gamma-1)/2} \right)^{2/(\gamma+5)},$$

- We also have: $E_B = \frac{\gamma+1}{4} E_{\mathrm{CR}}, \quad E_{\min} = \frac{\gamma+5}{\gamma+1} E_B$

General approach

- Things get messier!

$$\kappa = I\left(\frac{\sqrt{E_{\rm inj}^2 + 2m_e c^2 E_{\rm inj}}}{m_e c^2}\right)\left(\frac{\Gamma\left(\frac{3-\gamma}{2}\right)\Gamma\left(\frac{\gamma-2}{2}\right)}{2\sqrt{\pi}(\gamma-1)}\right)^{-1}$$

$$+ \frac{1}{\sum_i Z_i \nu_i}\left(\frac{m_p}{m_e}\right)^{2-\gamma}\left(\frac{2m_p c^2 E_{\rm inj}}{E_{\rm inj}^2 + 2m_e c^2 E_{\rm inj}}\right)^{(\gamma-1)/2}$$

$$\times \left(\sum_i A_i^{(3-\gamma)/2}\nu_i - \frac{1}{2(3-\gamma)}\left(\frac{2E_{\rm inj}}{m_p c^2}\right)^{(3-\gamma)/2}\right.$$

$$\left.\times \left(\frac{\Gamma\left(\frac{3-\gamma}{2}\right)\Gamma\left(\frac{\gamma-2}{2}\right)}{2\sqrt{\pi}(\gamma-1)}\right)^{-1}\right) - 1.$$

- For more details check

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MODIFIED EQUIPARTITION CALCULATION FOR SUPERNOVA REMNANTS

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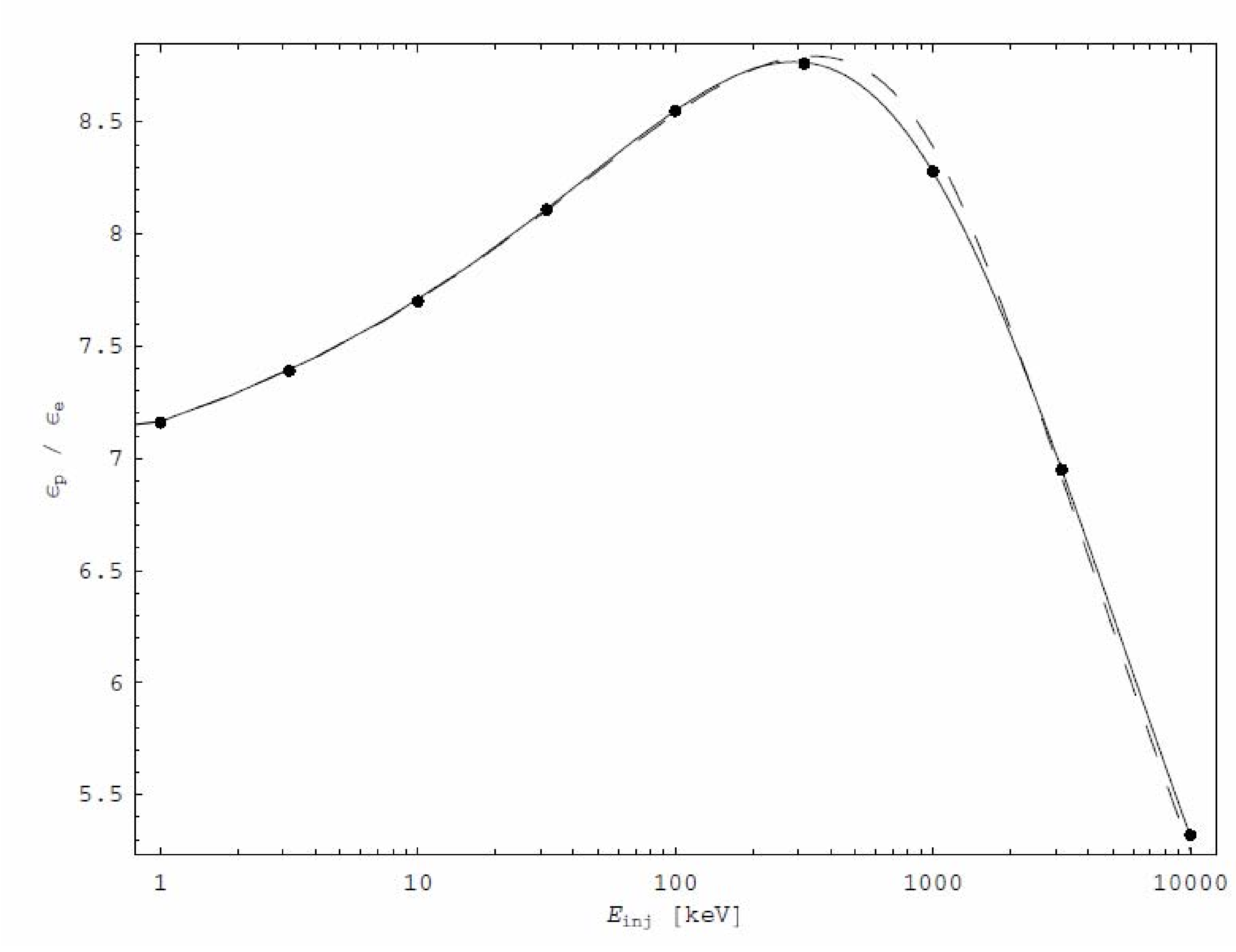
ABSTRACT

Determination of the magnetic field strength in the interstellar medium is one of the more complex tasks of contemporary astrophysics. We can only estimate the order of magnitude of the magnetic field strength by using a few very limited methods. Besides the Zeeman effect and Faraday rotation, the equipartition or minimum-energy calculation is a widespread method for estimating magnetic field strength and energy contained in the magnetic field and cosmic-ray particles by using only the radio synchrotron emission. Despite its approximate character, it remains a useful tool, especially when there are no other data about the magnetic field in a source. In this paper, we give a modified calculation that we think is more appropriate for estimating magnetic field strengths and energetics in supernova remnants (SNRs). We present calculated estimates of the magnetic field strengths for all Galactic SNRs for which the necessary observational data are available. The Web application for calculation of the magnetic field strengths of SNRs is available at http://poincare.matf.bg.ac.rs/~arbo/eqp/.

Key words: ISM: magnetic fields – ISM: supernova remnants – radio continuum: general

- Proton to electron energy ratio



- user-friendly PHP code:

  http://poincare.matf.bg.ac.rs/~arbo/eqp/

- corrected magnetic field:

$$\epsilon_B/\epsilon_{\mathrm{CR}} = \beta = \mathrm{constant}$$

$$B' = \left(\frac{4\beta}{\gamma + 1}\right)^{2/(\gamma+5)} B,$$

Conclusions

- Our approach is similar to Beck & Krause (integration over energies), however:

  1) we integrate over momentum to obtain energy density of particles (Beck & Krause assumed a break in the spectrum and described it with two different power laws)

  2) we take into account different ion species

  3) we use flux density at a given frequency and assume isotropic pitch angle distribution for the SNR as a whole

  4) we use energy ratio between ions and electrons (Pacholczyk used the same for protons and electrons), while Beck & Krause used ratio of the number density between protons and electrons.

- Our estimates of the magnetic field strengths are close to the values previous obtained by Beck & Krause (higher than those obtained by using Pacholczyk calculation)
- Work for the future:

  1) including the case $\alpha = 0.5$

  2) radio-evolution of supernova remnants


Thank you!